to elements of body area, to the specification of heat fluxes, and to surface normals.

Let $X'Y'Z'$ be a coordinate system fixed with respect to an orbiting body, so that, at perigee and with no displacements, X' coincides with X, Y' with Y, and Z' with Z. Then, the angles α,β, and γ define the orientation of any element surface area normal with respect to both primed and unprimed axes. When such a body is displaced or rotating in its orbital path, the orientation of an element surface area normal remains fixed with respect to the X',Y',Z' axes, and changes in the same manner as these axes change with respect to the X,Y,Z axes. Thus, by specifying the change between the primed and unprimed axes, one also specifies the change in orientation of an element surface area normal with respect to the unprimed coordinate system.

The orientation of the primed axes to the unprimed axes at any instant of time can be written with vector notation† in the general form

$$\alpha_d = \text{arc}\cos[\cos\alpha_0 l_{XX'} + \cos\beta_0 l_{XY'} + \cos\gamma_0 l_{XZ'}] \quad (1)$$

$$\beta_d = \text{arc}\cos[\cos\alpha_0 l_{YX'} + \cos\beta_0 l_{YY'} + \cos\gamma_0 l_{YZ'}] \quad (2)$$

$$\gamma_d = \text{arc}\cos[\cos\alpha_0 l_{ZX'} + \cos\beta_0 l_{ZY'} + \cos\gamma_0 l_{ZZ'}] \quad (3)$$

where subscript 0 refers to the angles α,β, and γ, defined at perigee with respect to the unprimed axes, in the case in which no yaw, pitch, or roll occurs. Subscript d refers to the angles α,β, and γ (referenced to the unprimed axes) when these angles change because of body rotation. $l_{XX'}$, $l_{YY'}$, $l_{ZZ'}$, etc., are the dot products $\bar{\imath}\cdot\bar{\imath}',\bar{\jmath}\cdot\bar{\jmath}', \bar{k}\cdot\bar{k}'$, etc. More specifically, $l_{XX'}$, $l_{YY'}$, etc., are the direction cosines between the primed and unprimed axes.

To be directly applicable to specific problems, it is necessary to evaluate the expressions $l_{XX'}, l_{YY'}$, etc., as functions of yaw, pitch, and continuous body roll (spin). Since the resultant body position may not be always independent of the order in which displacements are made, it is necessary to specify a convention making any resultant body position always independent of any order of displacement. This is as follows:

1) Yaw, represented by the angle ρ, always is taken about the unprimed X axis.

2) Pitch, represented by the angle K, always is taken about the Z' axis; the Z' axis forms a right-hand system with the X' and Y' axes. The Z' axis is coincident with the Z axis at perigee, when the body has no yaw, pitch, or roll.

3) Spin, represented by the angle W, always is taken about the Y' axis; the Y' axis is the longitudinal body axis and is coincident with the Y axis at perigee, with no yaw, pitch, or roll.

4) The angle W is the total angle rolled through in a period of time, t sec, i.e., $W = W_t \times t$, where W_t = angular spin rate in degrees per second and t = time in seconds.

5) All rotations about axes follow the standard right-hand-advancing-screw convention, except for pitch, which is taken in a sense opposite to that convention.

Following the foregoing convention, Eqs. (1–3) now may be written in a form applicable to any specific problem. The notation is the same as was used previously:

$$\alpha_d = \text{arc}\cos[\cos\alpha_0 \cos K \cos W + \cos\beta_0 \sin K + \cos\gamma_0 \cos K \sin W] \quad (4)$$

$$\beta_d = \text{arc}\cos[-\cos\alpha_0(\cos\rho \sin K \cos W - \sin\rho \sin W) + \cos\beta_0 \cos\rho \cos K - \cos\gamma_0(\sin\rho \cos W + \cos\rho \sin K \sin W)] \quad (5)$$

$$\gamma_d = \text{arc}\cos[-\cos\alpha_0(\sin\rho \sin K \cos W + \cos\rho \sin W) + \cos\beta_0 \sin\rho \cos K + \cos\gamma_0(\cos\rho \cos W - \sin\rho \sin K \sin W)] \quad (6)$$

† Equations (1–3) are taken from p. 154 of Ref. 2.

References

[1] Katz, A. J., "Determination of thermal radiation incident upon the surfaces of an earth satellite in an elliptical orbit," Inst. Aerospace Sci. Paper 60-58 (1958).

[2] Housner, G. W. and Hudson, D. E., *Applied Mechanics—Dynamics* (D. Van Nostrand Co. Inc., Princeton, N. J., 1958), p. 154.

Expanding Flow Ionization Nonequilibrium: Its Contribution to MHD Generators

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Nomenclature

A = nozzle cross-section area
T = temperature, °K
l = nozzle throat radius divided by $\tan\theta$
u = velocity
x = axial distance from nozzle throat toward nozzle exit
α = rate coefficient for cesium atoms and electrons, cm^3/molecule-sec
γ = electron density divided by gas density
ρ = gas density
θ = half-angle of the asymptotic cone of the nozzle

Subscripts

eq = at equilibrium
T = at nozzle throat
x = at position x in nozzle

ESCHENROEDER[1] has surmised that significant performance gain for MHD generators can result from the nonequilibrium behavior of a seeding material, cesium ion, in expanding flow. Effectively, potential energy might be converted to kinetic energy without a large loss in ion concentration and conductivity before the fluid enters the generator. Shorter generators would be possible, and the shortened length itself would tend to minimize concentration relaxation.

The purpose of the following is to report a particular application of the foregoing surmise to present-day MHD generators in order to determine whether the nonequilibrium conductivity can be large enough to require inclusion in generator power calculations. The finding is that, in regimes of flow applicable to MHD generators, contribution of nonequilibrium ionization can be neglected.

Conditions of temperature and pressure purposely were chosen which are somewhat too severe for present-day continuous MHD generator operation and therefore are among the most conducive to levels of ionization nonequilibrium which may affect generator performance. These combustor and nozzle entrance conditions are stoichiometric combustion of propane/oxygen at 30 atm and 3560°K. The combustion products, including cesium seed material, undergo isentropic expansion through an axisymmetric, hyperbolic nozzle down to 2 atm static pressure at the MHD generator entrance.

A mathematical development of ionization nonequilibrium of cesium in expanding flow of combustion products is given by Eschenroeder.[1] It finds expression as a nonlinear differential equation for one-dimensional steady flow:

$$d\gamma/dx = -(\gamma^2 - \gamma_{eq}^2)(\rho\alpha/u)$$

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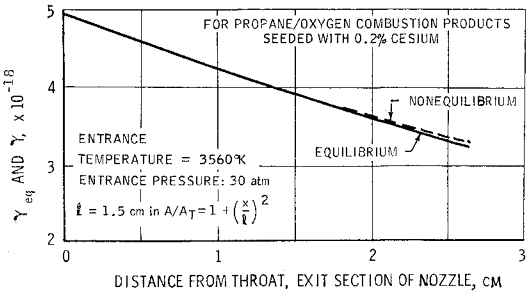

Fig. 1 Comparison of equilibrium and nonequilibrium ionization in expanded flow through an axisymmetric, hyperbolic nozzle

Here γ and γ_{eq} are, respectively, nonequilibrium and equilibrium electron densities divided by gas density, and α is the recombination coefficient for cesium ions and electrons. A value of rate coefficient $\alpha = 10^{-7}(250/T)^{3/2}$ was used. This is the lowest value of rate coefficient given by Eschenroeder and, accordingly, produces maximum ionization nonequilibrium. Values of $(\gamma_{eq})_x$ were obtained from the Saha equation for degree of ionization as a function of temperature and pressure.

Figure 1 compares γ and γ_{eq} as obtained. Ionization due to nonequilibrium is negligible under conditions that purposely were taken as extreme for continuous MHD generator operation. Little or no benefit is to be derived from this type of ionization nonequilibrium in present-day MHD generator design.

Reference

[1] Eschenroeder, A. Q., "Ionization nonequilibrium in expanding flows," ARS J. **32**, 196–203 (1962).

Concave Surfaces in Free Molecule Flow

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Nomenclature

m = molecular mass
N_i = total incident molecular flux
N_b = incident molecular flux from multiple reflections
N_∞ = incident molecular flux from freestream
r = radius of cylindrical arc surface
r_{12} = separation of surface elements $d\Sigma_1$, $d\Sigma_2$
R = gas constant (per unit mass)
S = molecular speed ratio = $U_\infty/(2RT_\infty)^{1/2}$
T_∞ = freestream temperature
T_b = temperature of surface
U_∞ = freestream velocity
z = spanwise coordinate
α = angle of incidence
θ = polar coordinate
Θ = limit on θ
ρ_∞ = freestream density

In two recent papers Chahine[1,2] has studied the aerodynamic characteristics of surfaces with cylindrical and spherical curvature, concave to a hyperthermal free molecule flow. It is the purpose of this note to point out that Chahine's results for the drag of the cylindrical surface are in error, and to give the corrections. An alternative approach to the problem used by the present author[3] has proved advantageous in the axisymmetric case in that the numerical evaluation of the results is relatively straightforward, whereas Chahine's results are left in the form of intractable quadruple integrals.

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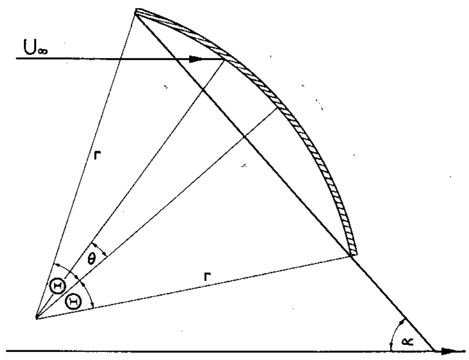

Fig. 1 Geometry of cylindrical surface

Following Chahine, consider an arc, infinite in extent, of a circular cylindrical surface (Fig. 1). The generators of the surface are normal to a free molecule flow, and no part of the surface is shielded from the freestream. In this paper the mean thermal velocity of the gas molecules in comparison with the freestream velocity is neglected, and a perfectly diffuse reflection of molecules from the surface, with perfect thermal accommodation, is assumed.

The momentum exchange normal to the surface per unit area per unit time may be expressed as

$$p(\theta) = p_\infty(\theta) + p_b(\theta) + p_r(\theta) \tag{1}$$

where p_∞ is the pressure component due to incident freestream molecules, p_b is that due to multiple reflections of molecules from other parts of the surface, and p_r is the component due to re-emission of molecules from the surface. Similarly, the tangential momentum exchange is

$$\tau(\theta) = \tau_\infty(\theta) + \tau_b(\theta) \tag{2}$$

Since re-emission is diffuse, $\tau_r = 0$. The freestream and re-emission terms take the well-known forms[4]

$$p_\infty(\theta) = \rho_\infty U_\infty^2 \sin^2(\alpha - \theta) \tag{3}$$

$$p_r(\theta) = mN_i(\theta)(\tfrac{1}{2}\pi RT_b)^{1/2} \tag{4}$$

$$\tau_\infty(\theta) = \rho_\infty U_\infty^2 \sin(\alpha - \theta)\cos(\alpha - \theta) \tag{5}$$

The multiple reflection terms remain to be determined.

Integration of the normal momentum components of the re-emitted molecules over all possible directions of re-emission shows that Eq. (4) is compatible with a mean velocity of re-emission given by

$$c = \tfrac{3}{4}(2\pi RT_b)^{1/2} \tag{6}$$

where the cosine law of diffuse reflection is taken to hold. In order to obtain the multiple reflection terms, one assumes that the molecules are all emitted from the surface with this mean velocity. From the geometry of Fig. 2 one finds that the number of molecules emitted by a surface element $d\Sigma_2$

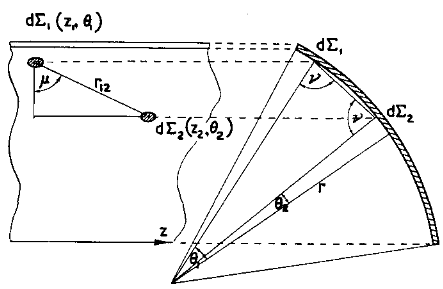

Fig. 2 Geometry of multiple reflection process